Frozen ghosts in thermal gauge field theory

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We review an alternative formulation of gauge field theories at finite temperature where unphysical degrees of freedom of gauge fields and the Faddeev-Popov ghosts are kept at zero temperature.

1.1. Introduction

Thermal gauge field theory is a combination of two difficult areas of physics, and so it is no surprise that some of its aspects are subtle. An apparently simple and rather powerful formalism is in common use,[1,2] which introduces propagators, a version of Wick's theorem, and even what appear to be quantum states, all of which are thermal generalisations of their zero-temperature counterparts. While many things may be calculated from this formalism, sometimes it does not apply,[3] and sometimes it is needlessly complicated.[4] It can be valuable, therefore, to go back to first principles, rather than making use of the conventional formalism without thinking. Doing so, we shall find that it in the real-time formalism it is possible and often advantageous to keep the tree-level propagators of ghosts and unphysical degrees of freedom free from thermal modifications in arbitrary linear

[1]Contribution to "Fundamental Interactions—A Memorial Volume for Wolfgang Kummer", D. Grumiller, A. Rebhan, D.V. Vassilevich (eds.)

gauges including covariant gauges, whereas in the standard formulation this is the case only for noncovariant gauges without propagating ghosts, such as the axial gauge.[5]

Already at zero temperature, there are two approaches to deriving the gauge-field-theory formalism. The first works with field operators and commutation relations, and introduces a space of kets. Some of these kets do not correspond to physical states, because the fields have unphysical degrees of freedom. It is necessary, therefore, to identify a subset of the kets corresponding to the physical states, most simply those that contain no scalar or longitudinal gauge particles. However, as was first noticed by Feynman,[6] unless one introduces additional ghost fields and the resulting kets, the probability of scattering from a physical state to an unphysical one is not zero. That is, the ghosts are needed to ensure that the S matrix is unitary within the subspace of physical states.

The other approach, which uses path integrals, does not explicitly consider states and the ghosts have to be introduced for an apparently very different reason. One can show that the two approaches are equivalent of course, but to do so is not simple: one has to introduce the BRS operator.[7,8] The operator approach is closer to the physics and so it is the one we use here.

At nonzero temperature, the propagators acquire a thermal part that has to be added to the zero-temperature Feynman propagator. As we shall discuss, there are two formalisms:

- All components of the gauge field, and the ghosts, become heated to the temperature T.
- Only the two physical degrees of freedom of the gauge field (the transverse polarisations) acquire the additional thermal propagator; the other components of the gauge field, and the ghosts, remain frozen at zero temperature. (This is for the bare propagators; self-energy insertions in the unphysical bare propagators do depend on the temperature.)

The second of these is the less commonly used, but in practice it is sometimes much simpler to apply. We shall describe it here. But before that, we go back to basics and remind ourselves of just what thermal field theory is trying to achieve.

1.2. Basics of equilibrium thermal field theory

For definiteness, consider QCD in Feynman gauge, though the discussion of any gauge theory in any covariant gauge will be similar. A system in thermal equilibrium is not in any particular quantum state; all one knows is the probability of it being in any one of a complete set of physical states. That is, one describes the system through a density matrix that expresses the knowledge that it is in thermal equilibrium:

$$\rho = Z^{-1}\mathbb{P}\exp(-H/T) \tag{1.1}$$

Here the units are such that Boltzmann's constant $k_B = 1$, H is the Heisenberg-picture Hamiltonian. $\mathbb{P}$ is a projection operator onto a complete set of physical states; we may choose to express it in terms of a complete orthonormal set of asymptotic in-states:

$$\mathbb{P} = \sum_i |i \text{ in}\rangle\langle i \text{ in}| \tag{1.2}$$

Z is called the grand partition function and is defined so as to make ρ have unit trace:

$$Z = \mathrm{tr}\ \mathbb{P}\exp(-H/T) \tag{1.3}$$

A trace is invariant under a change of the basis of states used to calculate it: any complete orthonormal set of states may be used and it may or may not include unphysical states, because their contribution is removed by $\mathbb{P}$.

1.3. Freezing unphysical degrees of freedom in the real-time formalism

Thermal field theory with gauge fields is more complicated than for scalar fields largely because of the presence of $\mathbb{P}$. The theory for scalar fields relies for its comparative simplicity on the commutativity of traces, tr AB = tr BA, but it is usually not true that tr $\mathbb{P}AB$ = tr $\mathbb{P}BA$.

Note that the states $|i \text{ in}\rangle$ are ordinary zero-temperature states, and the fields used to construct H are ordinary zero-temperature operators. The temperature T comes in only in that it weights the way that the states are combined together to construct the density matrix: from (1.1) and (1.2)

$$\rho = Z^{-1}\sum_i |i \text{ in}\rangle\langle i \text{ in}|\exp(-H/T) \tag{1.4}$$

Because we have chosen to express ρ in terms of asymptotic in states, to pick out a complete set of physical states we need to consider only noninteracting fields.

In the real-time formalism one can switch off interactions adiabatically at $t_i \to -\infty$, so that nonabelian gauge theories reduce to (a number of) noninteracting abelian ones. Here t_i refers to the point in time where the interaction-picture operators of perturbation theory coincides with the full Heisenberg operators.

One can then reduce the interaction-picture gauge fields to physical ones by projecting to transverse modes according to

$$A^\mu_{\text{phys.}}(k) = T^{\mu\nu}(k) A_\nu(k) \tag{1.5}$$

with

$$T^{0\mu} = 0, \quad T^{ij} = -(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}). \tag{1.6}$$

The unphysical fields are given by

$$A^\mu_{\text{unphys.}}(k) = (g^{\mu\nu} - T^{\mu\nu}(k))\, A_\nu(k), \tag{1.7}$$

and the ghost fields $\bar{c}$, c.

With this decomposition the corresponding parts in the free Hamiltonians commute and one can factorise

$$\begin{aligned} &\sum_i \langle i \text{ in}| e^{-\beta H_{0I}} \cdots A_{\text{phys.}} \cdots A_{\text{unphys.}} \cdots \bar{c} \cdots c \cdots |i \text{ in}\rangle \\ =& \sum_i \langle i \text{ in}| e^{-\beta H_{0I}^{\text{phys.}}} \cdots A_{\text{phys.}} \cdots |i \text{ in}\rangle \\ &\quad \times \langle 0| \cdots A_{\text{unphys.}} \cdots \bar{c} \cdots c \cdots |0\rangle \end{aligned} \tag{1.8}$$

where H_{0I} is the free Hamiltonian in the interaction picture and $|i \text{ in}\rangle$ are states obtained by acting exclusively with operators for the physical fields onto the vacuum state.[a] This leads to a perturbation theory where only the propagator for the physical gauge field $A^\mu_{\text{phys.}}$ is thermal and all other propagators remain as at zero temperature.

[a]It is of course still true that there is a many-one correspondence between physical states and the kets that represent them. There is also still the issue of indefinite metrics and negative-norm states. The probability of scattering into any given unphysical state is in fact not zero, but it is cancelled by the probability of scattering into other unphysical states.

In the real-time formalism, propagators have a 2×2 matrix structure, which (with the Schwinger-Keldysh choice of complex time path) reads

$$iD^{\mu\nu}(x) = \begin{pmatrix} \langle \mathrm{T} A^\mu(x) A^\nu(0) \rangle & \langle A^\mu(0) A^\nu(x) \rangle \\ \langle A^\mu(x) A^\nu(0) \rangle & \langle \tilde{\mathrm{T}} A^\mu(x) A^\nu(0) \rangle \end{pmatrix} \tag{1.9}$$

where T and $\tilde{\mathrm{T}}$ refer to time and anti-time ordering, respectively. In particular, a massless scalar (momentum-space) propagator reads

$$iD = M \begin{pmatrix} \frac{i}{k^2 - i\epsilon} & 0 \\ 0 & \frac{-i}{k^2 + i\epsilon} \end{pmatrix} M \tag{1.10}$$

with

$$M = \sqrt{n(|k_0|)} \begin{pmatrix} e^{\beta |k_0|/2} & e^{-\beta k_0/2} \\ e^{\beta k_0/2} & e^{\beta |k_0|/2} \end{pmatrix}, \tag{1.11}$$

where n is the Bose-Einstein distribution function. When only transverse gauge field modes have a nontrivial density matrix, this matrix structure applies only to the $T^{\mu\nu}$ projection of the gauge field propagator, whereas its complement is to be taken at zero temperature. So the latter as well as the ghost propagator involves the zero-temperature limit of the matrix M,

$$M_0 = \begin{pmatrix} 1 & \theta(-k_0) \\ \theta(k_0) & 1 \end{pmatrix}. \tag{1.12}$$

In Feynman gauge, the gauge field propagator thus reads

$$D^{\mu\nu} = -T^{\mu\nu} D - (g^{\mu\nu} - T^{\mu\nu}) D_0, \tag{1.13}$$

which can be easily generalized[11] to arbitrary linear gauges by replacing $g^{\mu\nu}$ in the above expression by the corresponding Lorentz structure appearing in the zero-temperature propagator.

Using these propagators, one can rather easily verify explicitly the gauge fixing independence of hard thermal loops[9,10] and also of the thermodynamic potential at the multi-loop level.[4]

However, a subtlety appears in applications of the hard-thermal-loop resummation program. Upon resummation of hard thermal loops, the gauge field propagator has not only physical poles corresponding to transverse polarizations, but also a collective mode with spatially longitudinal polarization.[12] One can show[11] that after resumming the hard-thermal-loop self-energy, the spatially longitudinal propagator component acquires the usual matrix structure of a propagator at finite temperature, which is in fact necessary so that no pinch singularities appear at higher orders of the loop expansion.

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